Application of Markov Drift Processes to Logistical Systems Modeling

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Scientists who contributed to problem early:
Sevastyanov (1962), Kosten (1974,1986), Wijngaard (1979), Anick, Mitra, and Sondhi (1982), Mitra (1988), Mitra and Mitrani (1991), Prabhu (1980, 1998), Postan (1989,1992,1998, 2006)

Geographical position of Ukraine

Odessa National Maritime University

- ONMU was founded in1930

- 8 Faculties and 5500 students
- Bachelors in Engineering, M.Sc. and PhD studies

Definition of Markov Drift Process

$(Z(t), \xi(t))$

$Z(t)$ is discrete component;

$\xi(t)$ is n-dimensional random walk.

The phase space $\boldsymbol{\Omega} = \boldsymbol{D} \times \boldsymbol{C}$, where $\boldsymbol{D}$ is a discrete set; $\boldsymbol{C}$ is a continuous set, $\boldsymbol{C}$

$\subseteq R_+^n, R_+^n$

is non-negative orthant of n-dimensional Eucledian space.

The basic differential equation

$$\xi'(t) = v_{Z(t)}(\xi(t)) \qquad \text{a.s.}, \qquad (1)$$

where

$v_{ki}(x), k = 1, 2, \ldots, n; i \in D, x \in C$

are the given bounded functions describing the "drift" of $\xi(t)$.

Specification of equation (1):

a)

$$C=[0,E_1]\times[0,E_2]\times\ldots\times[0,E_n];$$

b)

$$\xi_k'(t)=\sum_{i\in D} v_{ki} I(Z(t)=i)-\sum_{i\in D_k^-} v_{ki} I(Z(t)=i,\xi_k(t)=0)-$$

$$-\sum_{i\in D_k^+} v_{ki} I(Z(t)=i,\xi_k(t)=E_k),k=1,2,\ldots,n, \qquad \text{a.s.,} \qquad (2)$$

where *I*(*A*) is the indicator of an event *A*;

$$D_k^-=\{i:v_{ki}<0,i\in D\}\neq\varnothing, D_k^+=\{i:v_{ki}>0,i\in D\}\neq\varnothing,$$

$$\sup_i\left|v_{ki}\right|<l_k<\infty, k=1,2,\ldots,n.$$

The basic system of differential equations and boundary conditions for the case *n*=1

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)q_i(x,t)=-\lambda_i(x)q_i(x,t)+$$

$$+\sum_{j\neq i}\lambda_{ji}(x)q_i(x,t),\quad 0<x<E,\ i\in D, \qquad (1)$$

$$v_iq_i(0,t)=\sum_{j\in D^-\cup D^0}\lambda_{ji}(-0)p_j^-(t), i\in D^+, \qquad (2)$$

$$\frac{d}{dt}p_i^-+v_iq_i(0,t)=-\lambda_i(-0)p_j^-(t)+\sum_{j\in D^-\cup D^0;\, j\neq i}\lambda_{ji}(-0)p_j^-(t),$$

$$i\in D^-\cup D^0,$$

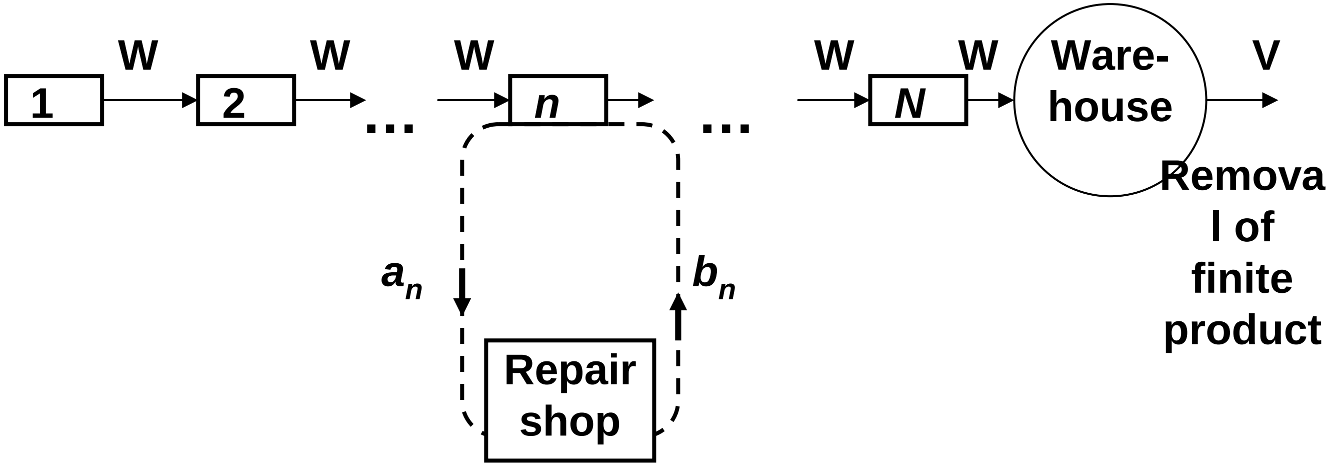
$$-v_iq_i(E,t)=\sum_{j\in D^+\cup D^0}\lambda_{ji}(E+)p_j^+(t), i\in D^-, \qquad (3)$$

$$\frac{d}{dt}p_i^+(t)-v_iq_i(E,t)=-\lambda_i(E+)p_j^+(t)+\sum_{j\in D^+\cup D^0;\, j\neq i}\lambda_{ji}(E+)p_j^+(t),$$

$$i\in D^+\cup D^0,$$

$$\sum_{i\in D^-\cup D^0}p_i^-(t)+\sum_{i\in D^+\cup D^0}p_i^+(t)+\sum_{i\in D}\int_0^E q_i(x,t)dx=1, \qquad (4)$$

Scheme of production line with unreliable units and storage of final product



W is production rate of line;

V is rate of removal of finite product from warehouse;

a_n rate of failures flow;

b_n rate of repairs completion flow.

System of differential equations and boundary conditions for determination of stationary distribution for production line with unreliable units

$$D=\{0,1,\ldots,N\},D^{-}=\{1,2,\ldots,N\},D^{+}=\{0\},D^{0}=\varnothing,$$

$$v_i=W-U,i\in D^{+};v_0=-U.$$

Differential equations:

$$Vq_0'(x)=-aq_0(x)+\sum_{n=1}^{N} b_n q_n(x),$$

$$-Uq_i'(x)=-b_i q_i(x)+a_i q_0(x),0<x<E,i\in D.$$

Boundary conditions:

$$Vq_0(0)=\sum_{n=1}^{N} b_n p_n^{-},$$

$$Uq_i(0)=b_i p_i^{-},i\in D^{-},$$

$$Uq_i(E)=a_i p_0^{+},i\in D^{-},$$

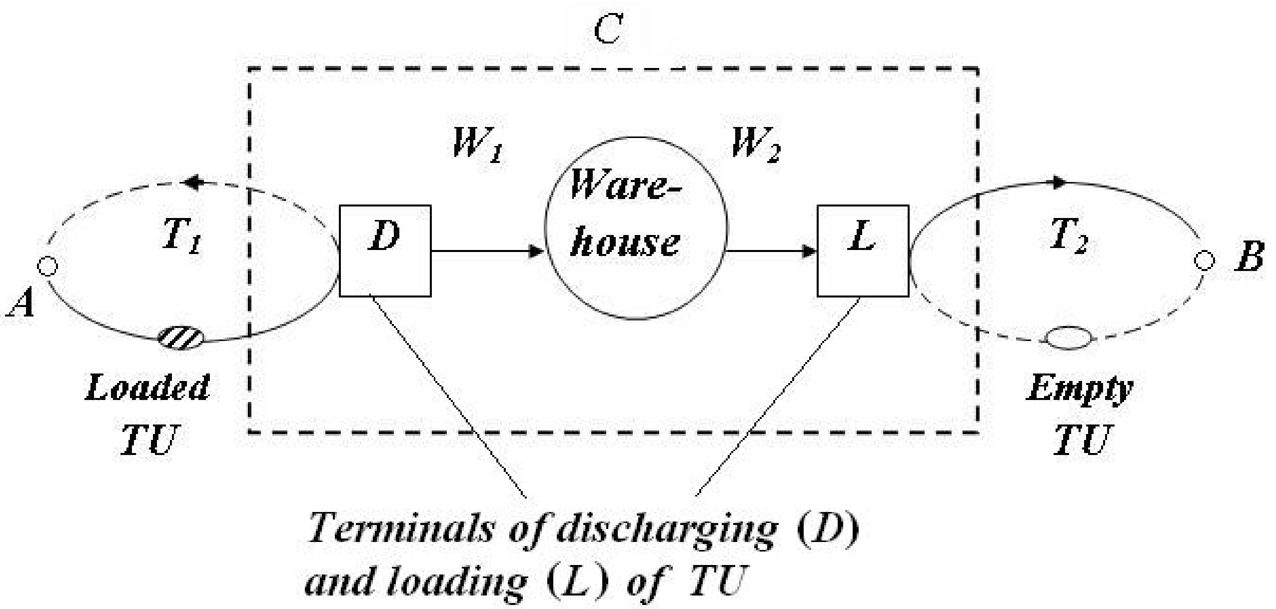
$$Vq_0(E)=ap_0^{+}.$$

Condition of normalization:

$$p_0^{+}+\sum_{n=1}^{N} p_n^{-}+\int_0^E \sum_{n=1}^{N} q_n(x)dx=1.$$

Here $V=W-U>0;a=a_1+a_2+\ldots+a_{N;}$

Scheme of interaction of transport units at point of transshipment Point of transshipment



W_1 is processing rate of unloading of TU;
W_2 is processing rate of loading of TU;
T_1, T_2 are the race durations for both TU.

System of differential equations and boundary conditions for determination of stationary distribution of interaction of two transport units via warehouse

D ={(0,0), (1,0),(0,1),(1,1)}, D^-={(0,1)}, D^+={(1,0),(1,1)},D^0={(0,0)}

$v_{10}=W_1$, $v_{11}=W_1-W_2>0$, $v_{01}=-W_2$, $v_{00}=0$

System of differential equations:

$$
\begin{aligned}
&0=-(a_1+a_2)q_{00}(x)+b_1q_{10}(x)+b_2q_{01}(x),\\
&W_1q'_{10}(x)=-(a_2+b_1)q_{10}(x)+b_2q_{11}(x)+a_1q_{00}(x),\\
&-W_2q'_{01}(x)=-(a_1+b_2)q_{01}(x)+b_1q_{11}(x)+a_2q_{00}(x),\\
&Vq'_{11}(x)=-(b_1+b_2)q_{11}(x)+a_1q_{01}(x)+a_2q_{10}(x),\ 0<x<E.
\end{aligned}
$$

Boundary conditions:

$$W_2 q_{01}(0) = a_1 p_{01}^-, \quad V q_{11}(0) = a_1 p_{01}^-, q_{10}(0) = 0,$$
$$W_2 q_{01}(E) = b_3 p_{11}^+, \; -W_1 q_{10}(E) = -a_2 p_{10}^+ + p_{11}^+,$$
$$-V q_{11}(E) = -(b_2 + b_3) p_{11}^+ + a_2 p_{10}^+.$$

Condition of normalization:

$$p_{01}^- + p_{10}^+ + p_{11}^+ + \int_0^E [q_{00}(x) + q_{01}(x) + q_{10}(x) + q_{11}(x)]dx = 1.$$

Here: $V=W_1-W_2$, $b_1=W_1/g_1$, $b_2=W_2/g_2$, $b_3=W_2/g_1$.

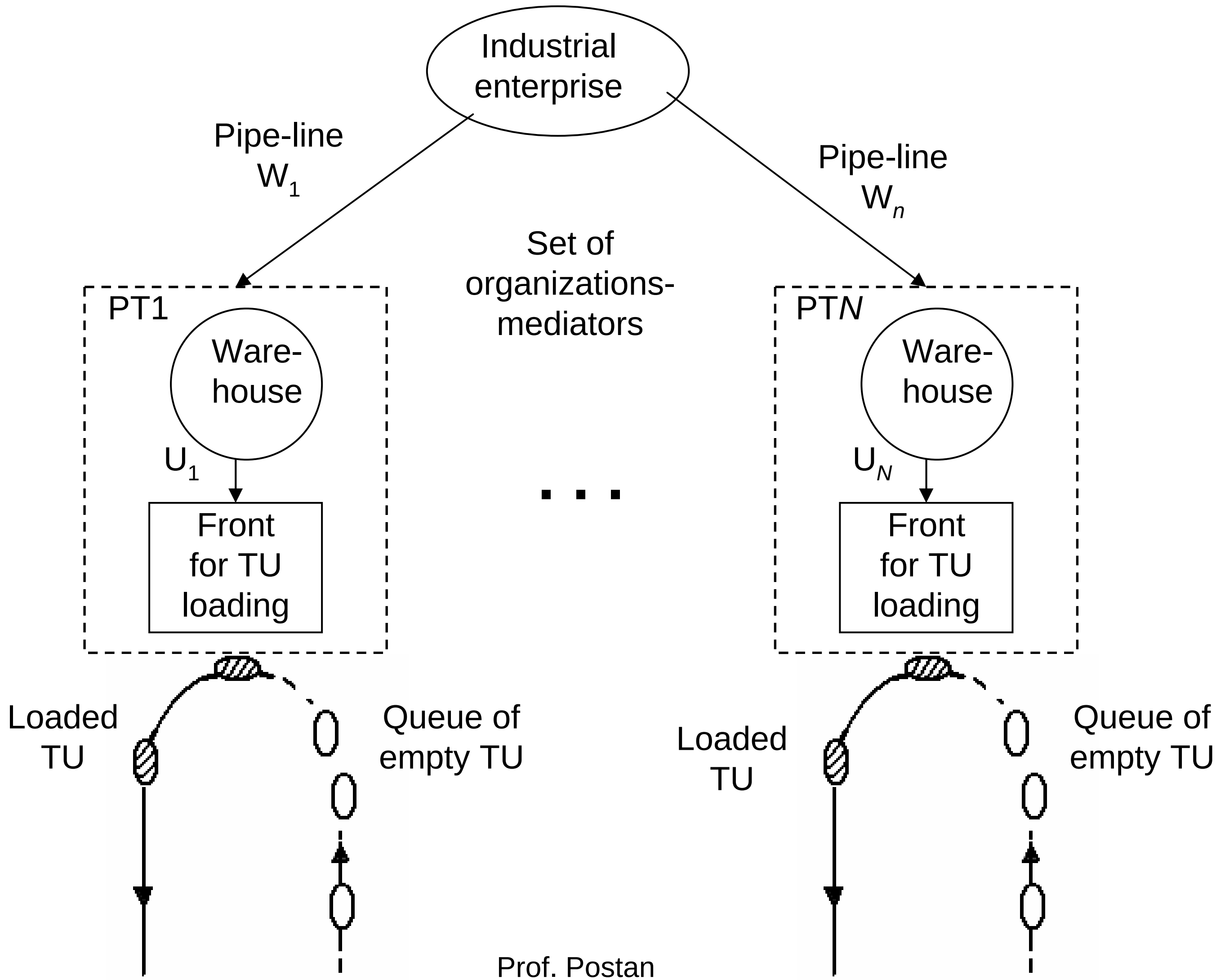
Main indices of interaction efficiency :

a)probability of transport units demurrage because of absence of another transport unit (p_{10}^+)

b)mean amount of cargo at warehouse

$$\mathrm{E}\xi = \int_0^E x[q_{00}(x) + q_{01}(x) + q_{10}(x) + q_{11}(x)]dx + E(p_{10}^+ + p_{11}^+).$$

Scheme 0f logistical system of liquid product distribution among points of transshipments



Optimal distribution of cargo-flows among a set of transshipment points

Objective function: total mean current profit of all organizations-mediators

$$\overline{P} = \sum_{n=1}^{N} (\pi_n W_n - c_n^{st} \mathrm{E}\xi_n) \to \max, \qquad (1)$$

where $\pi_n = \Pi_n - p_n - c_n$;

Π_n is the price for 1 *t* of product for customer at *n*-th PT;

p_n is the price for 1 *t* of product for *n*-th mediator;

c_n are transportation costs for delivery of 1 *t* of product to *n*-th PT;

c^{st}_n are storage expences per time unit for 1 *t* of product at *n*-th PT;

W_n is rate of product's coming at *n*-th PT from enterprise (control parameter);

$\mathrm{E}\xi_n$ is mean amount of product at warehouse at *n*-th PT.

Constraint:

$$\sum_{n=1}^{N} W_n \le W_0, \qquad (2)$$

where W_0 is production capacity of enterprise.

Solution of optimization problem (1), (2) for the case $E=\infty, R=0$,

$$E\xi_n = \frac{b_n W_n^2}{a_n[a_n U_n - (a_n + b_n) W_n]}, \quad a_n U_n > (a_n + b_n) W_n :$$

the optimal values of $\{W_n,\ n=1,2,\ldots,N\}$ are given by formulas

$$W_n = \frac{a_n U_n}{a_n + b_n}\left[1 - \sqrt{\frac{b_n c_n^{st}}{a_n (a_n + b_n)(\pi_n + \varphi) + b_n c_n^{st}}}\right], n = 1,2,\ldots,N,$$

where φ is Lagrange multiplier which satisfies the equation

$$\sum_{n=1}^{N} \frac{a_n U_n}{a_n + b_n} \sqrt{\frac{b_n c_n^{st}}{b_n c_n^{st} + a_n (a_n + b_n)(\pi_n + \varphi)}} = \sum_{n=1}^{N} \frac{a_n U_n}{a_n + b_n} - W_0 > 0.$$

Thank you for your attention

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